2023 SUMMER SCHOOL ON DIFFERENTIAL GEOMETRY

Complex Geometry

1. Let ω_{FS} be the Fubini–Study metric on $\mathbb{CP}^n$.
(1) Compute the expression of ω_{FS} using local coordinates.
(2) Compute the Ricci form $\mathrm{Ric}(\omega_{FS})$.
(3) When $n = 1$, compute the integral $\int_{\mathbb{CP}^1} \omega_{FS}$.

2. Let $\mathcal{F}$ be a sheaf of commutative rings on a topological space X and $s \in \mathcal{F}(X)$. Show that the set

$$\{x \in X | s_x \in \mathcal{F}_x^*\}$$

is open in X, where $\mathcal{F}_x^*$ denote the group of units of the stalk $\mathcal{F}_x$ (namely $\mathcal{F}_x^*$ consists of all the invertible elements in the ring $\mathcal{F}_x$).

3. Let $\Omega \subset \mathbb{C}^n$ be a domain. Let $f \in \mathcal{O}^*(\Omega)$.
   (1) Show that $\log |f|^2$ is a subharmonic function on Ω.
   (2) If $|f|^2$ is a constant, show that f must be constant.

4. Let X be a compact complex manifold. Assume that L is an ample line bundle on X. Show that there exists a smooth Hermitian metric h on L with positive Chern curvature.

5. Let $\mathcal{O}(1)$ be the dual of the tautological line bundle on $\mathbb{CP}^n$. Compute $\mathrm{vol}(\mathcal{O}(1))$.